Magnetoresistance in semiconductor structures with hopping conductivity: Effects of random potential and generalization for the case of acceptor states

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We reconsider the theory of magnetoresistance in hopping semiconductors. First, we have shown that the random potential of the background impurities affects significantly the pre-exponential factor of the tunneling amplitude which becomes short range in contrast to the long-range one for purely Coulomb hopping centers. This factor to some extent suppresses the negative interference magnetoresistance and can lead to its decrease with temperature decrease which is in agreement with the earlier experimental observations. We have also extended the theoretical models of positive spin magnetoresistance, in particular, related to presence of doubly occupied states (corresponding to the upper Hubbard band) to the case of acceptor states in two-dimensional structures. We have shown that this mechanism can dominate over the classical wave-shrinkage magnetoresistance at low temperatures. Our results are in semiquantitative agreement with the experimental data.

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I. INTRODUCTION

The problem of magnetoresistance in the hopping transport was addressed decades ago. In particular, an interest to this topic was related to important additional information provided by corresponding experiments (including estimates of the localization length). The most general and natural mechanism of positive magnetoresistance of orbital nature is related to the shrinkage of the localized wave function by magnetic field; it was extensively reviewed in Ref. 1. Another important mechanism of orbital magnetoresistance was considered by Nguyen *et al.*[2] (for the detailed review see Ref. 3). It is related to the presence of the underbarrier scattering of hopping electrons by intermediate hopping sites and to interference between different hopping trajectories. Note that for the effective interference the difference of lengths of different trajectories should not exceed the localization length which restricts the location of the trajectories to the so-called "cigar region." A significance of this mechanism was emphasized by the exponentially broad scatter of hopping probabilities corresponding to different "hopping resistors." As a result of "logarithmic averaging" over different configurations the most important role is played by those interference patterns where the total hopping probability almost vanishes as a result of destructive interference. The magnetic field suppresses interference and thus the average effect is *negative* magnetoresistance that appears to be linear in a weak magnetic field (although becoming quadratic at $H \rightarrow 0$). Important features of the approach discussed in Ref. 3 were as follows. First, the authors exploited an assumption of the presence of many intermediate scatterers. Second, following the theory,[4] the authors assumed the pre-exponential factor to be equal to μ/r, where μ is the scattering amplitude and r is the distance between the hopping site and the scattering center. The picture of interference magnetoresistance considered in Ref. 3 was very rich including change in the sign of magnetoresistance, effects of spin glass, etc. (see also Refs. 5–8). This negative magnetoresistance was later observed in a number of experimental works (see, for example, Refs. 9–21).

Somewhat later the problem was also discussed in Refs. 22 and 23 where it was noticed that in realistic situations the number of intermediate scatterers is small and most probably equal to one or (in average) even less. Another important ingredient of the paper[23] was the usage of wave functions typical for Coulomb centers which have not contained pre-exponential dependence on r. In contrast to the "scattering states" of Ref. 3 which contained pre-exponential factors decaying with r, this situation can be specified as a "strong scattering case." Note that, although in Ref. 23 the authors considered two-dimensional (2D) hopping, they addressed to the case of delta-doped layer and thus the asymptotic of the wave functions was similar to three dimensions (3D). An important result of the theory suggested in Ref. 23 was the following: the patterns of the negative magnetoresistance were almost universal predicting the maximum value of ~ 0.6 of the total resistance, and even the combination of the negative magnetoresistanse and positive wave-shrinkage magnetoresistance gave the maximum value of negative peak (with respect to average resistance) of around 40%.

Unfortunately, these predictions for strong scattering case were not in a good agreement with the experiment. First, in most of experimental studies the effect of negative magnetoresistance has not exceeded 10% and typically was around several percent. Then, it was shown[13] that in 3D semiconductors the negative magnetoresistance is suppressed with decrease in temperature after the crossover from Mott-type hopping (at higher temperatures) to Efros-Shklovskii (ES) hopping over the states within the Coulomb gap. The suppression of negative magnetoresistance at low temperatures was later confirmed in Ref. 15. In the paper[13] we explained such a behavior as a result of decrease in concentration of the scattering centers within the Coulomb gap. However our calculations were based on assumption that the pre-exponential factor of the wave functions asymptotic corresponded to scattering states of Ref. 3 (weak-scattering case) rather than to hydrogenlike asymptotics exploited in Ref. 23. Later on[14] we have also demonstrated that to fit the experimental data one should also take into account spin mechanisms of magnetoresistance. The first one, considered in Ref. 24, is based

on the fact that an intermediate scatterer should be occupied to produce a negative scattering amplitude. Thus the interference depends on the mutual orientation of the spin of hopping electron and of spin of scattering center. Without external magnetic field only one half of the configurations gives an interference. In magnetic field all localized spins are aligned which increases the role of interference and, correspondingly, leads to increase in resistance.

Another spin mechanism of positive magnetoresistance was first considered in Ref. 25 and then was studied in detail in Ref. 26. It is related to a presence of doubly occupied hopping sites (corresponding to the upper Hubbard band). Due to spin correlations on these sites requiring *s* pairing of the spins (recall that we consider here an electron rather than hole hopping) some hopping transitions are suppressed in magnetic field (like ones from single-occupied site to another single-occupied site).

As mentioned above, the incorporation of all of the relevant factors allowed us to reach quantitative agreement between the theoretical model and experimental data. However basing the scattering state asymptotic we exploited an assumption of correlated impurity configurations which had no solid theoretical proof.

Another important request to the theory of hopping magnetoresistance was related to 2D hopping. As mentioned above, the theoretical model of Ref. 23 exploited 3D localized wave functions which does not hold for typical experiments for doped quantum wells where the wave functions have 2D character. Then, we should mention a new important experimental results[18,19] obtained for selectively doped quantum well structures where both centers of the wells and centers of the barriers were doped ensuring formation of the upper Hubbard band. These structures demonstrated suppression of negative magnetoresistance with decrease in temperature for the samples with a higher degree of doping. Although we attempted to explain this behavior in a similar way as for 3D structures in Ref. 13, it hardly works because of important difference between 2D and 3D physics.

In what follows we will give a consistent description of magnetoresistance in both 3D and 2D structures including different orbital and spin mechanisms. An important conclusion of ours is that in most occasions one deals with a weak-scattering case rather than with a strong scattering case. If we are restricted to the lower Hubbard band, the decisive factor is related to the presence of charged centers outside of the cigar region not involved into interference. The random potential imposed by these centers restricts the extension of the hydrogenlike asymptotics of the scattering centers up to the distance to the closest charged center while outside this region the pre-exponential of the asymptotics appears to be similar to the one for the potential-well case (weak-scattering limit). For the case of the states within the upper Hubbard band an additional factor is related to the non-Coulombic potential of the scattering center which is also of a short-range character. The resulting picture of hopping magnetoresistance appears to be different from the one suggested in Ref. 23 (based on the pure Coulomb wave functions) and from the one of Ref. 3 (exploiting the assumption of a large number of intermediate scatterers). We also emphasize a role of spin mechanisms of positive magnetoresistance which can dominate over the wave-shrinkage magnetoresistance at low temperature. In this concern a special analysis is given to spin mechanisms for acceptor centers which have an important differences with respect to the earlier discussed case of donor impurities.

II. NEGATIVE MAGNETORESISTANCE IN 3D CASE

Let us consider negative magnetoresistance in 3D case. As mentioned above, an important ingredient to be included with respect to the previous studies is a random potential imposed by the intermediate charged centers (including both donors and acceptors).

We shall start from a solution of the Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\Psi + U_0(\mathbf{r})\Psi + U(\mathbf{r})\Psi = E\Psi. \tag{1}$$

Here $U_0(\mathbf{r})$ is the potential of impurity ($U_0=-\alpha/r$ in the case of a hydrogenlike impurity level), $U(\mathbf{r})$ is the random potential that comes from the charged centers mentioned above, m is the electron mass in the conduction band (or the hole mass in the valence band), and E is the exact electron energy [we consider $|E| \gg U(\mathbf{r})$].

Because of the fact that typical hopping lengths are much larger a, the characteristic localization length, we can solve (1) for $r \gg a$. Moreover, the typical hopping length r_h appears to be much larger than the typical distance between charged centers which can be roughly estimated as $n^{-1/3}$, where n is the dopant concentration. Indeed, for 3D variable range hopping of Mott type

$$r_h = \xi a, \quad \xi = \left(\frac{T_0}{T}\right)^{1/4}, \tag{2}$$

where

$$T_0 \simeq \frac{21}{g a^3}, \tag{3}$$

where $g \sim n/\mathcal{E}_B$ is the density of states and $\mathcal{E}_B$ being the Bohr energy. Thus one obtains

$$r_h n^{1/3} \sim \left(\frac{21 a n^{1/3} \mathcal{E}_B}{T}\right)^{1/4} \tag{4}$$

that is even for the Mott law $r_h n^{1/3}$ is bigger than 1. The more so, it holds for a Coulomb gap regime where r_h strongly exceeds the corresponding values for the Mott regime.

Thus we are interested in asymptotics of the wave functions at distances much larger than $n^{1/3}$ which for moderately compensated material can be considered as the correlation length of random potential imposed by the charged centers. If so, we can make an important conclusion. Namely, the random potential U is formed by long-range Coulomb centers and in this sense the potential produced by the "parent" (for the considered wave function) impurity at distances larger than $n^{-1/3}$ makes no difference with potential produced by other charged centers. In other words, for $r > n^{-1/3}$ one should not discriminate between U and U_0 and should assume that the resulting potential U has a spatial average equal to zero.

Having in mind that $r \gg a$ we approach the problem of the asymptotics of the wave function by means of the WKB method. We introduce the function ϕ with regard of the normalization factor for the function Ψ as $\Psi=(\pi a^3)^{-1/2}\exp(-\phi/\hbar)$. The Shrödinger equation in 3D case leads to

$$\phi'^2 - \hbar(\phi'' + 2\phi'/r) = 2m[U(\mathbf{r}) + |E|]. \tag{5}$$

We will expand this equation into series with respect to $\hbar \to 0$. In the zeroth order we have

$$\phi_0' = \sqrt{2m(U+|E|)}. \tag{6}$$

This order gives us the exponent. To get the pre-exponent factor we should use the first order of perturbation theory. Here we have in mind that function ϕ_0' at large r is actually a constant—the more so that its linear expansion in $U(r)$ is averaged out,

$$\phi_1' = \frac{\hbar}{r}. \tag{7}$$

Accordingly the expression for ϕ up to the first order is

$$\phi = \int \sqrt{2m(U+|E|)}dr + \hbar \ln r/r_{min}, \tag{8}$$

where $r_{min} \sim n^{-1/3}$. And finally the wave function Ψ is

$$\Psi = \exp\left(-\frac{1}{\hbar}\int \sqrt{2m(U+|E|)}dr\right)\frac{r_{min}}{r(\pi a^3)^{1/2}}. \tag{9}$$

Having in mind the considerations given above, we can average

$$(1/\hbar)\int \sqrt{2m(U+|E|)}dr = kr,$$

where

$$k = (1/\hbar)\langle\sqrt{2m(U+|E|)}\rangle.$$

Note that k differs from $k_0=\sqrt{2m|E|}/\hbar$ only in the second order in $U/|E|$ ($\propto U^2/E^2$), as the mean value $\langle U\rangle$ is zero. We also neglect U in the pre-exponent factor.

So at distances from the scattering center larger than the correlation length of random potential (assumed to be equal to the average distance between charged centers) the wave-function asymptotics has a pre-exponential factor $\propto r^{-1}$ which agrees with the scheme exploited in Refs. 3, 13, and 14 for 3D hopping.

Now, following approaches[3,13] let us estimate the hopping probability between the sites 1 and 2 in the presence of an intermediate "scattering center" with regard that the energies of the centers obey relation $|E_3| \gg |E_1|, |E_2|$:

$$P \propto |J_1 + J_2|^2, \quad J_1 = I_{12}, \quad J_2 = -\frac{I_{13}I_{32}}{|E_3|}. \tag{10}$$

Here J_1 and J_2 are the hopping amplitudes related to direct and scattered path correspondingly. Note that the destructive interference (leading to negative magnetoresistance) implies that $E_3<0$) which means that in equilibrium the scattering site is occupied.

The energy overlapping integrals are

$$I_{ij} = \mathcal{E}_B\frac{r_{min}}{r_{ij}}\exp(-r_{ij}/a), \tag{11}$$

where we have assumed that $r_{ij} > r_{min} = n^{-1/3}$; $\mathcal{E}_B$ being the Bohr energy. Without magnetic field this amplitudes are real. Though in magnetic field their phases are different and the hopping probability is

$$P \propto |J_1 + J_2 e^{i\varphi}|^2. \tag{12}$$

Here phase difference $\varphi = 2\pi\Phi/\Phi_0$, where Φ is the magnetic flux through the surface bounded by hoping paths. Φ_0 is the elementary magnetic flux. Accordingly, the interference magnetoresistance for $\varphi < 1$ is

$$\ln\frac{r(H)}{r(0)} \propto -\left\langle \int dE_3 g(E_3) \int \ln\left[1 + J_1(J_1 - J)\frac{\varphi^2}{J^2}\right]d^3r_3\right\rangle. \tag{13}$$

Here $J=J_1+J_2$, g is the density of states, and r_3 is the scatterer position. The angular brackets correspond to ensemble average. We consider magnetoresistance to be determined over hops with small J, so we neglect the term J_1/J in Eq. (13) and get

$$\ln\frac{r(H)}{r(0)} \propto -\left\langle \int dE_3 g(E_3) \int \ln\left[1 + \frac{J_1^2\varphi^2}{J^2(r_3,E_3)}\right]d^3r_3\right\rangle. \tag{14}$$

To obey $J_1 \simeq J_2$ one, first, should have $r_{12} \simeq r_{13} + r_{23}$ with an accuracy of order of localization length a. Then, having in mind the pre-exponential factors one notes that for small r_{min} the only possibility to satisfy the relation is to have one of the distances, r_{13} or r_{23} to be small. We will assume that it holds for r_{23} which is estimated as

$$r_{23} \sim r_{min}\frac{\mathcal{E}_B}{E_3}. \tag{15}$$

Let us choose the surface at which $J=0$ and transform the integration over r_3 in the way $d^3r_3 \to d^2\mathcal{R}dR_\perp$, where $\mathcal{R}$ is the coordinate on the surface in question while $R_\perp$ is the coordinate along the normal to the surface where we assume that $R_\perp=0$ corresponds to $J=0$. In the lowest order in $R_\perp$ we have $J=(dJ/dR_\perp)R_\perp$. As it is seen, the integration of the logarithm term over $R_\perp$ gives

$$\frac{\varphi J_1}{dJ/dR_1}.$$

Finally, the integration of

$$\frac{J_1}{dJ/dR_\perp}$$

over $d^2\mathcal{R}$ gives approximately the volume accessible for the site 3. Note that we have $r_{13}+r_{23} \le r_{12}+a$, thus the projection of $\mathbf{r_{23}}$ on the plane normal to $\mathbf{r_{12}}$ should be less than $(r_{23}a)^{1/2}$. As a result, the integration over the spatial coordinate $\mathbf{r_3}$ gives

$$\sim \frac{r_{min}^2 \mathcal{E}_B^2}{E_3^2} a\varphi. \tag{16}$$

In its turn, the area of the interference loop (entering the estimate of φ) is

$$r_{12}\left(r_{min}\frac{\mathcal{E}_B}{E_3}a\right)^{1/2} \tag{17}$$

Note that these estimates actually hold for all accessible values of r_{23} up to $r_{23} \sim r_{12}/2$. The final result depends on the behavior of $g(\varepsilon)$. For g=const (a Mott-type hopping) the integration over E_3 is naturally controlled by the lower limit which accordingly to Eq. (15) corresponds to the larger possible value of $r_{23} \sim r_{12}/2$. In this case the right-hand side (rhs) of Eq. (13) is $\propto r_h^{5/2}$, where $r_h \sim r_{12}$.

In contrast, for the Coulomb gap hopping the integration over E_3 is controlled by the upper level, E_C, corresponding to the edge of Coulomb gap. In this case r.h.s. of Eq. (13) is $\propto r_h$ since the value of r_{23} does not depend on r_h.

Now let us consider a combination of NMR with a positive magnetoresistance (PMR) related to wave-function shrinkage which can be estimated as

$$\ln\frac{\rho(H)}{\rho(0)} = \left(\frac{H}{B}\right)^2, \tag{18}$$

where

$$B^2 = \frac{\alpha c^2 \hbar^2}{r_h^3 a e^2}. \tag{19}$$

Here α is a numerical parameter resulting from he percolation theory; for Mott-type hopping $\alpha \sim 400$ (Ref. 1) while for the Coulomb gap hopping different sources give $\alpha \sim 300$ and $\alpha \sim 700$.

In its turn, NMR can be rewritten as

$$\ln\frac{\rho(H)}{\rho(0)} = k\frac{H}{B}, \tag{20}$$

where

$$k = g_M \mathcal{E}_B r_{min} r_h 2a\alpha^{1/2} \quad \text{Mott law},$$

$$k = \frac{\kappa^3}{e^6}\frac{r_\Delta^{5/2}}{r_h^{1/2}} a E_C^3 2\alpha^{1/2} \quad \text{ES law}. \tag{21}$$

Here κ is the dielectric constant, r_Δ is the typical hopping length for the states corresponding to the edge of the Coulomb gap while E_C is the width of Coulomb gap. One sees that as a result we have minimum of resistance,

$$H_{min} = \frac{k}{2}B, \quad \ln\frac{\rho(H_{min})}{\rho(0)} = -\frac{k^2}{4}. \tag{22}$$

It is seen that the value of H_{min} decreases with temperature decrease irrespective to the type of the variable range hopping. At the same time for samples corresponding to Mott law the absolute value of resistance in minimum *increases* with temperature decrease while for the case of the Coulomb gap hopping it decreases with temperature decrease.

III. NEGATIVE MAGNETORESISTANCE IN 2D

Let us now approach the problem of negative magnetoresistance in a 2D structure where impurity wave functions are quantized in the orthogonal to impurity plane direction. First we will consider the case where we deal only with single-occupied or empty impurity centers (as it was done above for a 3D case).

Let us start with an approximation of 2D impurity wave function in the $r \gg a$ limit. Analogously to the previous case we neglect $U_0(\mathbf{r})$ and introduce function ϕ as $\Psi = (\pi a^2)^{-1/2}\exp(-\phi/\hbar)$. The corresponding WKB equation is

$$\phi'^2 - \hbar(\phi'' + \phi'/r) = 2m[U(\mathbf{r}) + |E|]. \tag{23}$$

Following the same procedure as for a 3D case we obtain

$$\Psi = \exp\left(-\frac{1}{\hbar}\int \sqrt{2m(U+|E|)}dr\right)\left(\frac{r_{min}}{r\pi a^2}\right)^{1/2}. \tag{24}$$

Analogously to 3D case this wave function is nearly equal to the potential-well wave function $\propto \exp(-kr)/\sqrt{r}$, where $k = \langle\sqrt{2m|E|+U}\rangle/\hbar$ which differs from $k_0 = \sqrt{2m|E|}/\hbar$ only in the second order in U/E.

Now let us consider negative magnetoresistance related to the interference contribution to the hopping probability. Following the same lines as for a 3D case we obtain an equation similar to 14 except that the integration is over d^2r_3 and the density of states g also corresponds to 2D. An important difference is related to the fact that now the value of J vanishes at

$$r_{23} = r_{min}(\mathcal{E}_B/E_3)^2. \tag{25}$$

With a similar transformation of the variables the integration of the logarithmic term over the coordinates gives

$$\left(\frac{r_{min}\mathcal{E}_B^2}{E_3^2}\right)^{3/2} a^{1/2}\varphi \tag{26}$$

while the effective loop area is

$$r_{12}\left[r_{min}\left(\frac{\mathcal{E}_B}{E_3}\right)^2 a\right]^{1/2}. \tag{27}$$

Since in 2D in the Coulomb gap regime $g \propto \varepsilon$ one notes that irrespectively to the hopping law the integration over E_3 is controlled by the lower possible values of E_3 leading finally to the estimates of $r_{23} \sim r_{12}$. Thus one obtains

$$k_2 = g_M \mathcal{E}_B r_{min}^{1/2} r_h 2a^{1/2}\alpha^{1/2} \quad \text{Mott law},$$

$$k_2 = \frac{\kappa^2}{e^4}\mathcal{E}_B^2 r_{min} r_h^{1/2} 2a^{1/2}\alpha^{1/2} \quad \text{ES law}. \tag{28}$$

Thus, as is seen, for the situation considered above for 2D the only combination of interference NMR and wave-shrinkage PMR cannot lead to a suppression of NMR with decrease in the temperature (at least for a low-temperature limit of linear NMR) since for both laws the temperature derivative of k stays negative.

An important feature of the 2D quantum well structure is an easy possibility to have an occupation of the upper Hub-

bard band. Namely, if we dope not only the well regions but also the barrier regions, the carriers from the barriers are captured by the wells and can form doubly occupied states. It was this situation which was realized in our experiments described in Refs. 18 and 19. Since in these experiments we dealt with GaAs/AlGaAs structures of quantum wells with p doping by Be, here we will also imply acceptor centers.

In our experiments the central regions of both wells and barriers were nearly equally doped by acceptor impurity Be. Thus the holes from the barriers have a possibility to occupy the second position for acceptor in the wells forming the upper Hubbard band. However for the hole there was another possibility—to stay around its native acceptor in the barrier forming a single-occupied center which we will denote as $\tilde{A}^0$. The corresponding scenario was first discussed in Ref. 27. As a result, at the Fermi level we have centers with different occupation numbers—at least, A^+ (doubly occupied). A^0 (single occupied) $\tilde{A}^0$ (holes bound to the barrier acceptor), and A^- (empty barrier acceptor with no hole around). Note that the structure of A^+ centers in quantum wells was first studied in Ref. 28.

The possibility for a hole to form A^+ or $\tilde{A}^0$ center depends on relation between the binding energies of these centers, U_b and $\tilde{U}_b$. In particular, if $U_b > \tilde{U}_b$, then all the barrier acceptors form A^- centers while all the acceptors in the well form A^+ centers. However for our experiments the distance between the barrier acceptor and the interface between the barrier and well was not large and we expect $\tilde{U}_b > U_b$. In this case the probability to form A^+ center depends on the distance between the barrier acceptor and the closest acceptor in the well. Indeed, the formation of A^+ center profit from the interaction between A^+ center and A^0 center.[27]

Here we assume that some holes from the barrier are still coupled to their parent acceptors ($\tilde{A}^0$ centers) and some are localized on the acceptors in the well (A^+ centers). According to charge conservation the number of $\tilde{A}^-$ centers (that are free $\tilde{A}^0$ centers) is equal to the number of A^+ centers.

$$N(A^+) = N(\tilde{A}^-). \tag{29}$$

In addition, we believe that there exists a random potential that overlap the energies of different types of centers. If the variances of $\tilde{A}^0$ and $\tilde{A}^-$ energies are equal, Eq. (29) leads to equal densities of states for $\tilde{A}^0$ and $\tilde{A}^-$ at the Fermi level. For our purpose we assume that this densities of states are at least comparable.

As for the negative magnetoresistance for the upper Hubbard band, it can be considered in the same way as for the lower Hubbard band discussed above. Note that the scattering potential strongly decays with distance $U_0 \propto r^{-4}$ and thus the corresponding asymptotics of the wave functions are similar to the one given by Eq. (24) but one should take $r_{min} = a$.

IV. SPIN MECHANISMS OF MAGNETORESISTANCE FOR ACCEPTOR STATES

We shall start from the mechanism of spin magnetoresistance first suggested in Ref. 24 which seems to be especially important for acceptor dopants. It is related to the fact that interference can occur only if the spin states of the final states for both tunneling paths coincide. For three-cite configuration we discuss it means that the initial and intermediate centers should have the same spin projections (we remind that for destructive interference in question the energy of intermediate center should be negative, i.e., at equilibrium this center should be occupied). For the case of acceptor states corresponding to the lower Hubbard band the corresponding configuration is in our case $\tilde{A}^0 - \tilde{A}^0 - A^-$ where the role of intermediate center is played by the site $\tilde{A}^0$. When dealing with holes one should take into account that the spin-orbit interaction that significantly complicates the problem. Namely, in general case the hole momentum does not conserve during the hop. We are grateful to A. S. Iosselevich who attracted our attention to this problem. However if we restrict ourselves to the case of thin quantum well and take momentum projection on the growth axis (we consider it to be also the axis of magnetic field) the conservation actually takes place and the problem can be treated similarly to the electron one.

The interference takes place when initial impurity of the hop and the scatterer have the same state (i.e., the same momentum projection on the axis of magnetic field). In thin quantum well the ground state of the acceptor is doubly degenerate at zero field. So at $H=0$ the interference probability is $P(H=0)=1/2$. However in strong magnetic field the holes momenta are aligned and do not affect (destructive) interference, that is, in this case $P(H \to \infty)=1$. Thus an increase in the magnetic field leads to enhancement of destructive interference which means positive magnetoresistance similar to the one discussed in Ref. 24.

This probability is different for upper Hubbard band. Actually in thin quantum well the ground state of A^+ center is not degenerate so when the hop is completely through upper Hubbard band states ($A^+ - A^+ - A^0$ hop) the interference probability is $P \equiv 1$. If the states of both the Hubbard bands coexist at the Fermi level, it can be estimated that the average statistical factor $P(H=0)$ is still on the order of 1/2, although its value at strong fields, $P(H \to \infty)$ appears to be somewhat smaller than unity.

At weak magnetic fields one expects a degree of spin alignment to be $\propto (\mu g H)^2/T^2$ and thus the statistical factor is equal

$$P(H) \simeq P(H=0) + \alpha \left(\frac{\mu g H}{T} \right)^2. \tag{30}$$

Here the coefficient α according to more detailed statistical calculations which we are going to present elsewhere can be estimated to be on the order of 1/7.

Since P describes the probability of destructive interference, one concludes that at weak fields the positive magnetoresistance resulting from statistical factor P is quadratic in terms of magnetic field. It can be estimated as follows:

$$\ln \frac{R(H)}{R(0)} = \alpha \left(\frac{\mu g H}{T} \right)^2 \frac{\Delta R_{sat}}{R(0)}, \tag{31}$$

where ΔR_{sat} is the saturation value for the interference contribution to resistance with no account of spin degrees of

freedom which is achieved when the phase φ in Eq. (12) exceeds 2π. As it can be estimated, the ratio $\Delta R_{sat}/R(0)$ is $\propto r_h$ for Mott hopping and $\propto r_h^{1/2}$ for the Coulomb gap hopping.

In its turn, it coexists with the linear negative magnetoresistance (of orbital nature) which at relatively weak fields can be estimated as

$$P(H=0)\Delta R_{sat}\frac{H}{H_{sat}}.$$

As is known, if at the Fermi level the states of the lower and the upper Hubbard bands coexist, there also exists a specific spin mechanism of positive magnetoresistance first considered in Ref. 25 (and later discussed in detail in Ref. 26) for n-type 3D structures. In such structures one deals with D^0 (occupied donors), D^- (doubly occupied donors), and D^+ (empty donors). Without external magnetic field the following configurations of hops are possible: $D^-\rightarrow D^0$, $D^0\rightarrow D^0$, $D^0\rightarrow D^+$, and $D^-\rightarrow D^+$. In the magnetic field the spins of D^0 centers are polarized and thus the hops $D^0\rightarrow D^0$ are forbidden (since in the final state of the second site corresponding to D^- the spins should be in opposite directions). In the same way the transitions $D^-\rightarrow D^+$ are also suppressed. Thus the resistance increases as a result of application of external magnetic field.

In our case of p-type structures the situation appears, again, more complicated due to a more complex structure of A^+ centers. However, in the limit of thin quantum well, the considerations given in Refs. 25 and 26 still hold. Basing on the calculations similar to given in Ref. 26 one obtains for the weak-field limit $\mu gH<T$ the following estimate:

$$\ln\frac{R(H)}{R(0)}=CF\left(\frac{g\mu_b H}{T}\right)^2, \tag{32}$$

where $C\sim 1$,

$$F=\frac{2g_l g_u}{(g_l+g_u)^2} \tag{33}$$

while g_l and g_u are the densities of states of the lower and upper Hubbard bands. Note that for a low concentration of dopants g_u is controlled by the concentration of A^+ centers while g_l—by the concentration of A^- centers and thus $g_l=g_u$. At stronger magnetic fields when $\mu gH>T$, the corresponding contribution to magnetoresistance still increases with magnetic field increase until μgH reaches the value ξT and then saturates.[25,26]

One notes that at low enough temperatures the positive magnetoresistance of the spin nature suggested in Ref. 25 can exceed the wave-shrinkage magnetoresistance. At the same time this contribution at relatively weak fields when $\mu gH<T$ is expected to be comparable to the spin magnetoresistance resulting from interference term discussed above. Summarizing both spin contributions to quadratic magnetoresistance we estimate the coefficient k resulting from a similar parametrization of the positive quadratic and linear negative magnetoresistance as was done above:

$$k_2=g_M E_B r_{min}^{1/2} r_h 2a^{1/2}\beta \quad \text{Mott law},$$

$$k_2=\frac{\kappa^2}{e^4}E_B^2 r_{min} r_h^{1/2} 2a^{1/2}\beta \quad \text{ES law},$$

$$\beta=P(H=0)\frac{T}{g\mu_B}\frac{r_h^{3/2}a^{1/2}e}{c\hbar}\left(CF+\alpha\frac{\Delta R_{sat}}{R(0)}\right)^{-1/2}. \tag{34}$$

Thus, as is seen, for the Mott case at $T\rightarrow 0$ $k\propto T^{1/3}$ while for the ES case it is $\propto T^{1/4}$.

Note that in our calculations we assumed that the value of H_{min} still corresponds to a linear behavior of negative magnetoresistance which means that the magnetic flux through the interference area is much less than the magnetic-flux quantum Φ_0. The critical field H_{sat} corresponding to a crossover from the linear behavior to saturation of the negative magnetoresistance is given as

$$H_{sat}\simeq\frac{\Phi_0}{2\pi r_h^{3/2}a^{1/2}}. \tag{35}$$

Correspondingly, if H_{min} given by Eq. (22) in appears to be larger than H_{sat} our calculations given above are invalid and one should compare the positive magnetoresistance with the saturated negative magnetoresistance rather than with the linear negative magnetoresistance. One notes that in contrast to linear magnetoresistance which is proportional to the area of the interference loop for the saturation magnetoresistance this proportionality is omitted. As a result, as it was noted above, the temperature dependence of the saturation value of negative magnetoresistance $\Delta R_{sat}/R(0)$ results from factors $\propto r_h$ for the Mott hopping and $\propto r_h^{1/2}$ for the Coulomb gap hopping. It is seen that the corresponding increase in the saturation magnetoresistance with temperature decrease is much weaker than increase in the positive magnetoresistance. Then, in the case $H_{min}>H_{sat}$ it is the value of H_{sat} which corresponds to minimal resistance since it separates the region of resistance decrease due to the negative magnetoresistance and resistance increase due to the positive magnetoresistance. However at this situation the spin magnetoresistance [Eq. (31)] is also saturated so the temperature behavior of positive magnetoresistance is related to Eq. (18) and (or) to Eq. (32).

In its turn let us consider the temperature behavior of the relation between H_{min} and H_{sat}. According to Eqs. (22) and (34),

$$H_{min}=2g_M E_B r_{min}^{1/2}a\frac{T^2}{(g\mu_B)^2 CF}\frac{r_h^{5/2}e}{c\hbar}\propto T^{7/6} \quad \text{(Mott law)},$$

$$H_{min}=2\frac{\kappa^2}{e^4}r_{min}a\frac{T^2}{(g\mu_B)^2 CF}\frac{r_h^2 e}{c\hbar}\propto T \quad \text{(ES law)}. \tag{36}$$

At the same time $H_{sat}\propto T^{1/2}$ for the Mott law and $H_s\propto T^{3/4}$ for the ES law. Thus the ratio H_{min}/H_{sat} decreases with temperature decrease and this decrease is more pronounced for the Mott law.

V. DISCUSSION

In Fig. 1 we present our experimental results from Ref. 13 for 3D hopping concerning the temperature behavior of mag-

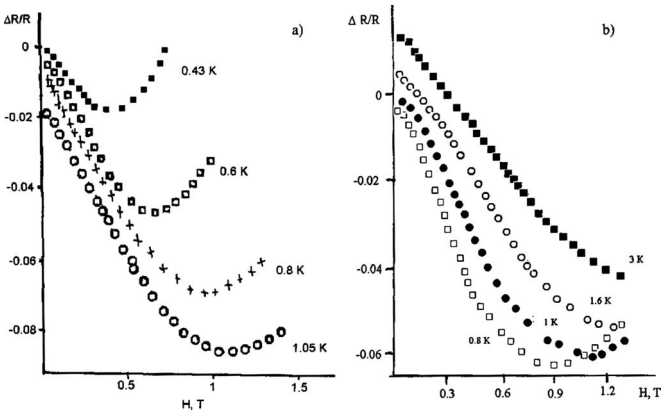

FIG. 1. Temperature behavior of magnetoresistance for bulk CdTe crystals doped by donor impurities (Cl). (a) The curves for the sample in the Coulomb gap regime and (b) for the sample in the Mott regime.

netoresistance for the regimes of Coulomb gap hopping [Fig. 1(a)] and of Mott-type hopping [Fig. 1(b)]. It is seen that these results are in a qualitative agreement with the predictions of Sec. II. In particular, the minimal value of resistance increases with temperature decrease for the Mott-type hopping and decreases for the Coulomb gap hopping. As was noted in Sec. I, the agreement was strongly improved when we had taken into account more subtle spin effects,[14] however here we will not go into these details discussed earlier.

At Figs. 2 and 3 we present experimental results described in Refs. 18 and 19 for multiple quantum well structures *p*-GaAs/AlGaAs:Be with different dopant concentration *n*. It is seen that for a sample with smaller concentration (Fig. 2) the negative magnetoresistance is strongly enhanced with temperature decrease while for the sample with larger dopant concentration (Fig. 3), in contrast, it is suppressed with a temperature decrease.

To our opinion, the difference of magnetoresistance curves for the samples with different *n* is related to the following fact. The sample with smaller concentration is far

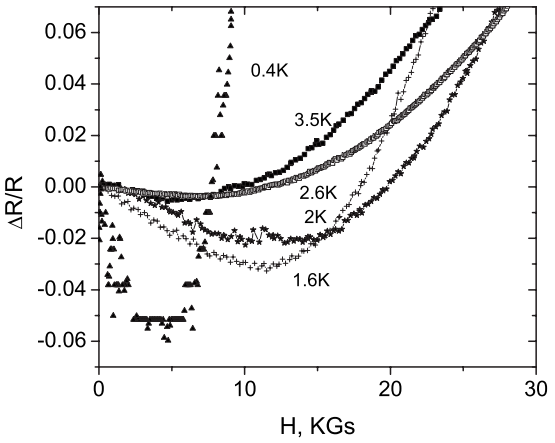

FIG. 2. Temperature behavior of resistance for the structures of ten GaAs wells of thickness 15 nm, separated by AlGaAs barriers with thickness 15 nm. The central parts of both wells and barriers were doped by *p*-type impurity Be with concentration 1×10^{17} cm^{-3}.

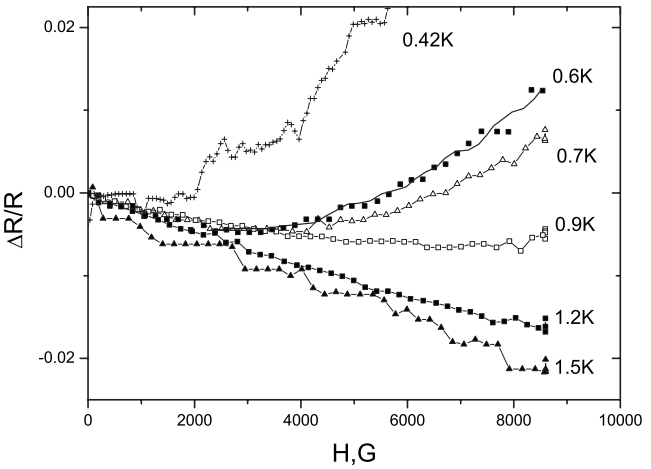

FIG. 3. Temperature behavior of magnetoresistance for the structures similar to described at Fig. 2 but with concentration of Be 9×10^{17} cm^{-3}.

from the metal-insulator transition and the localization length is relatively small, $a\sim10$ nm. Thus, in a view of small *n* and small *a* the three-cite approximation for interference contribution holds, $nr_h(ar_{min})^{1/2}\leq1$. For a heavily doped sample $a\sim20$ nm and $n\sim10^{12}$ cm^{-2}, correspondingly, $nr_h(ar_{min})^{1/2}>1$. As a result, the three-cite approximation for this sample does not hold and the interference loop includes large number of scatterers. As was noted above, the spin statistical factor for each additional site with nonzero spin at $H=0$ for acceptor impurities is $\sim1/2$. Correspondingly, the interference contribution for loops involving many intermediate scatterers vanishes at $H=0$. As a result, the linear contribution to negative magnetoresistance is, in any case, much smaller than for weakly doped samples. In contrast, the quadratic positive magneroresistance resulting from the statistical factor given by Eq. (31) strongly increases with temperature decrease,

$$\propto T^{-7/3} \quad \text{(Mott law)}, \quad \propto T^{-9/4} \quad \text{(ES law)}. \tag{37}$$

In addition, we can expect that for the sample with large *n* $H_{sat}<H_{min}$, it is H_{sat} which plays the role of H_{min}. Due to weak temperature dependence of R_{sat} the temperature behavior at the fields larger than H_{sat}, that is, corresponding to the minimum of $\rho(H)$, is completely controlled by the spin PMR which gives $\rho(H)\propto T^{-\alpha}$ with $\alpha>2$. Indeed, an increase in the resistance by factor of 4–5 is observed at the fields larger than 0.3 T for the temperature variation from 0.9 to 0.4 K.

VI. CONCLUSIONS

To conclude, we have reconsidered the existing theory of hopping magnetoresistance. We have shown that the random potential induced by the background impurities can affect the asymptotics of the localized states and, as a result, suppress to some extent the negative magnetoresistance related to

interference effects. We have also generalized the theory for the case of acceptor states in 2D structures including the effects of upper Hubbard band. The results obtained are in agreement with the existing experimental data. In particular, we explain the suppression of negative magnetoresistance with temperature decrease observed earlier for both 3D and 2D structures.

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